CBCS SCHEME

USN | | | | | | | | | |

**18MAT31**

## Third Semester B.E. Degree Examination, Jan./Feb. 2021
# Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

**Note:** ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

## Module-1

1 a. Find the Laplace transform of cos t cos 2t cos 3t. **(06 Marks)**

b. If $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ and $f(t + 2a) - f(t)$, show that $L\{f(t)\} = \dfrac{1}{s^2} \tan h \left(\dfrac{as}{2}\right)$. **(07 Marks)**

c. Find the Inverse Laplace transforms of :

i) $\dfrac{2s+1}{s^2+6s+13}$ ii) $\dfrac{1}{3}\log\left(\dfrac{s^2+b^2}{s^2+a^2}\right)$. **(07 Marks)**

**OR**

2 a. Express the function f(t) in terms of unit step function and find its Laplace transform, where

$$f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}.$$ **(06 Marks)**

b. Find the Inverse Laplace transform of $\dfrac{s^2}{(s^2+a^2)^2}$ using Convolution theorem. **(07 Marks)**

c. Solve by the method of Laplace transforms, the equation
$y'' + 4y' + 3y = e^{-t}$ given $y(0) = 0$, $y'(0) = 0$. **(07 Marks)**

## Module-2

3 a. Obtain the Fourier series of the function $f(x) = x^2$ in $-\pi \le x \le \pi$. **(06 Marks)**

b. Obtain the Fourier series expansion of

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ x - 2\pi, & \pi < x < 2\pi \end{cases}.$$ **(07 Marks)**

c. Find the Cosine half range series for $f(x) = x(\ell - x)$, $0 \le x \le \ell$. **(07 Marks)**

**OR**

4 a. Obtain the Fourier series of $f(x) = |x|$ in $(-\ell, \ell)$. **(06 Marks)**

b. Find the sine half range series for

$$f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - \pi, & \pi/2 < x < \pi \end{cases}.$$ **(07 Marks)**

c. Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table. **(07 Marks)**

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y | 9 | 18 | 24 | 28 | 26 | 20 |

## Module-3

5 a. If $f(x) = \begin{cases} 1-x^2 & , \ |x|<1 \\ 0 & , \ |x| \ge 1 \end{cases}$. Find the Fourier transform of f(x) and hence find value of $\int_0^\infty \frac{x\cos x - \sin x}{x^3} dx$. **(06 Marks)**

b. Find the Fourier Cosine transform of

$$f(x) = \begin{cases} 4x, & 0<x<1 \\ 4-x, & 1<x<4 \\ 0, & x>4 \end{cases}.$$ **(07 Marks)**

c. Find the Z – transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. **(07 Marks)**

**OR**

6 a. Solve the Integral equation

$$\int_0^\infty f(\theta)\cos\alpha\theta \, d\theta = \begin{cases} 1-\alpha & , \ 0 \le \alpha \le 1 \\ 0 & , \ \alpha > 1 \end{cases}$$ hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$. **(06 Marks)**

b. Find the Inverse Z – transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. **(07 Marks)**

c. Using the Z – transform, solve $Y_{n+2} - 4Y_n = 0$, given $Y_0 = 0$, $Y_1 = 2$. **(07 Marks)**

## Module-4

7 a. Using Taylor's series method, solve the Initial value problem $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$. Consider upto 4th degree term. **(06 Marks)**

b. Use modified Euler's method to compute y(0.1), given that $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ by taking $h = 0.05$. Consider two approximations in each step. **(07 Marks)**

c. Given that $\frac{dy}{dx} = x - y^2$, find y at x = 0.8 with

| | | | | |
|---|---|---|---|---|
| x : | 0 | 0.2 | 0.4 | 0.6 |
| y : | 0 | 0.02 | 0.0795 | 0.1762 |

By applying Milne's method. Apply corrector formula once. **(07 Marks)**

**OR**

8 a. Solve the following by Modified Euler's method $\frac{dy}{dx} = x + |\sqrt{y}|$, $y(0) = 1$ at $x = 0.4$ by taking $h = 0.2$. Consider two modifications in each step. **(06 Marks)**

b. Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$. Compute y(0.2) by taking h = 0.2 using Runge – Kutta method of order IV. **(07 Marks)**

c. Given $\frac{dy}{dx} = (1+y)x^2$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, determine y(1.4) by Adam's Bashforth method. Apply corrector formula once. **(07 Marks)**

## Module-5

9 a. Given y'' - xy' – y = 0 with y(0) = 1, y'(0) = 0. Compute y(0.2) using Runge – Kutta method. **(06 Marks)**

b. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

c. Prove that the geodesics on a plane are straight lines. **(07 Marks)**

**OR**

10 a. Find the curve on which functional $\int_0^1 [(y')^2 + 12xy]dx$ with y(0) = 0, y(1) = 1 can be extremized. **(06 Marks)**

b. Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ by computing the value of dependent variable corresponding to the value 1.4 of the independent variable by applying Milne's method using the following data. Apply corrector formula once. **(07 Marks)**

| | | | | |
|---|---|---|---|---|
| x : | 1 | 1.1 | 1.2 | 1.3 |
| y : | 2 | 2.2156 | 2.4649 | 2.7514 |
| y' : | 2 | 2.3178 | 2.6725 | 3.0657 |

c. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is Catenary $y = c\cosh\left(\frac{x+a}{c}\right)$. **(07 Marks)**

* * * * *

# CBCS SCHEME

USN | | | | | | | | | | |  **18CS32**

## Third Semester B.E. Degree Examination, July/August 2021
## Data Structures and Applications

Time: 3 hrs. Max. Marks: 100

***Note: Answer any FIVE full questions.***



1 a. Write the drawback of static memory allocation. Explain in detail the different functions of dynamic memory allocation. **(08 Marks)**
b. Write a program to search for key element in an array using binary search. **(07 Marks)**
c. Write the difference between structure and union. **(05 Marks)**

2 a. Write a program to sort integers in increasing order using selection sort algorithm. **(06 Marks)**
b. Write a function: (i) to find out the length of the string (ii) string concatenation, without using built-in function. **(08 Marks)**
c. Write a program to search for an element in the sparse matrix. **(06 Marks)**

3 a. What is stack? Write a program to implement push, pop and display operations for stacks using arrays. **(08 Marks)**
b. Convert the following infix expression to post fix expression:
(i) $((A + (B - C) * D) \wedge E + F)$ (ii) $X\$Y\$Z - M + N + P/Q$ **(06 Marks)**
c. Write a program to evaluate the postfix expression. **(06 Marks)**

4 a. Explain the drawback of ordinary queue. Write a program to implement push, pop and display operations for circular queue using array. **(08 Marks)**
b. Write a recursive program to find out the GCD of two numbers. **(05 Marks)**
c. What is double ended queue? Write a program to implement the same with required functions. **(07 Marks)**

5 a. Write a program to implement queue using singly linked list. **(08 Marks)**
b. Write a function to search for key element in a list using Singly Linked List. **(06 Marks)**
c. Write a function to delete a node based on information field using doubly Linked List. **(06 Marks)**

6 a. Write a function to count the number of nodes in the List. **(04 Marks)**
b. Write a function to perform the following using circular doubly Linked List with header node.
(i) insert_front → insert element at front end
(ii) delete_rear → delete element from rear end **(08 Marks)**
c. Write a function to add two polynomial using Linked List. **(08 Marks)**

7 a. Explain the following with suitable example:
(i) Binary tree
(ii) Binary search tree
(iii) Complete binary tree
(iv) Skewed tree **(08 Marks)**

b. Construct a tree using the given tree traversals:
in-order : GDHBAEICF
post-order : GHDBIEFCA (04 Marks)

c. Write a function to create and search for an element in binary search tree. (08 Marks)

**8** a. Write a program to insert an element in to binary tree. (08 Marks)

b. Write a function to traverse the tree using
(i) pre-order
(ii) post-order
(iii) in-order traversal (06 Marks)

c. Explain Threaded Binary Tree in detail. (06 Marks)

**9** a. Explain the different functions for file operations. (06 Marks)

b. Write a program to sort the array elements using radix sort. Show tracing to sort the given array elements increasing order using radix sort. 52, 43, 24, 67, 78, 96, 81, 63, 27. (08 Marks)

c. Write a function to sort the array elements in increasing order using insertion sort. (06 Marks)

**10** a. Write a program to print the reachable nodes of a graph from the source node using BFS method. (06 Marks)

b. Write the adjacency matrix and adjacency. List representation for the given graph in Fig.Q10(b).

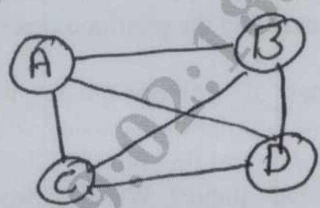

Fig.Q10(b) (06 Marks)

c. Explain hashing in detail. (08 Marks)

* * * * *

**CBCS SCHEME**

USN | | | | | | | | | | 18CS33

# Third Semester B.E. Degree Examination, July/August 2021
## Analog and Digital Electronics

Time: 3 hrs. Max. Marks: 100

**Note:** ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

**1** a. With a neat sketch, explain the construction and working of Light Emitting Diode (LED). **(06 Marks)**

b. For the given circuit in Fig.Q.1(b) Si transistor with $\beta = 50$, calculate the $I_B$, $I_C$ and $V_{CE}$. Draw the DC load line and determine the operating point. **(06 Marks)**

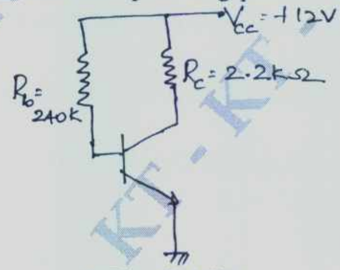


Fig.Q.1(b)

c. With a neat circuit diagram and waveform, explain the working of Astable multivibrator. **(08 Marks)**

**2** a. What is a filter? Compare between active filters and passive filters. **(06 Marks)**

b. With a neat diagram and waveform, explain working of relaxation oscillator. **(08 Marks)**

c. Explain the different components of regulated power supply. **(06 Marks)**

**3** a. Simply the given expression using K-map
$F(A, B, C, D) = \sum m(0, 1, 4, 8, 9, 10) + \sum d(2, 11)$ **(08 Marks)**

b. Using a prime-implicant charts, find all minimum SOP solution using Quine-Mc-Clusky method for $f(w, x, y, z) = \sum m(1, 3, 4, 5, 6, 7, 10, 12, 13) + \sum d(2, 9, 15)$. **(12 Marks)**

**4** a. Find all prime implicants of the following given function and find all minimum solutions using Petrick method.
$F(A, B, C, D) = \sum m(7, 12, 14, 15) + \sum d(1, 3, 5, 8, 10, 11, 13)$ **(12 Marks)**

b. Using the map-entered variable, use 4 variable maps to find the minimum SOP expression for the function
$G(A, B, C, D, E, F) = m_0 + m_2 + m_3 + Em_5 + Em_7 + Em_9 + m_{11} + m_{15} + d(1, 10, 13)$ **(08 Marks)**

**5** a. Write the truth table for the AND-OR functions for four valued simulations. **(06 Marks)**

b. With a suitable assumption, explain the timing diagram of an AND-OR circuit. **(06 Marks)**

c. What is hazard? Explain the different type of hazard with an example. **(08 Marks)**

**6** a. Explain multiplexer with an example. Realize the 8:1 multiplexer using 2:1 and 4:1 multiplexer. **(08 Marks)**

b. With a neat diagram, explain the 3 to 8 decoder. **(06 Marks)**

c. With a neat sketch, explain the structure of PLA. **(06 Marks)**

7 a. Explain the structure of an VHDL module. Write a VHDL code for 4:1 multiplexer. (08 Marks)
b. Write a program for the implementation of full-Adder using VHDL. (06 Marks)
c. With a neat diagram, explain switch debouncing circuit using an S-R latch. (06 Marks)

8 a. What is a flip flop? Explain the gated D-latch, with a neat diagram. (06 Marks)
b. Explain the Master-Slave J-K flip-flop with a neat diagram, using NAND gates. (10 Marks)
c. Explain T-flip flop with a diagram. (04 Marks)

9 a. What is Register? With a neat diagram, explain the register with data, load, clear and clock inputs. (08 Marks)
b. With a neat sketch, explain the working of Serial In Serial Out (SISO) Right shift register. (06 Marks)
c. What are the difference between the synchronous and Asynchronous counters? (06 Marks)

10 a. Design a synchronous counter for the sequence $0 \rightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 6 \rightarrow 7 \rightarrow 0 \rightarrow 3$ using J-K flip-flop. (10 Marks)
b. Design a counter using S-R flip-flop for the following given count $0 \rightarrow 4 \rightarrow 7 \rightarrow 2 \rightarrow 3 \rightarrow 0$. (10 Marks)

* * * * *

**CBCS SCHEME**

USN | | | | | | | | | | 18CS34

# Third Semester B.E. Degree Examination, July/August 2021
## Computer Organization

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. With neat block diagram, explain the basic operational concepts of a computer. **(05 Marks)**
b. Write basic performance equation and explain and define the terms involved in it. **(05 Marks)**
c. What is an Addressing mode? Explain the following addressing modes with one example for each : i) Absolute ii) Immediate iii) Indirect iv) Auto increment. **(10 Marks)**

2 a. Explain Big – Endian and Little – Endian assignment. Consider a computer that has a byte addressable memory organized in 32 words, according to Little – Endian scheme. A program reads ASCII characters entered at a keyboard and store them in successive byte location starting at 2000. Show how the contents of the three memory words at locations 2000, 2004 and 2008 after the string "VTU BELAGAVI" has been entered. (ASCII codes : V = 56H , T = 54H , U = 55H , " " = 20H , B = 42H , E = 45H , L = 4 CH , A = 41H , G = 47H , I = 49H). **(05 Marks)**
b. Register $R_1$ and $R_2$ of computer contain the decimal values 1200 and 4600. What is Effective Address (EA) of the memory operand in each of the following instructions?
i) Load 20 ($R_1$), $R_5$ ii) Move # 3000 , $R_5$ iii) Store 30 ($R_1$, $R_2$) , $R_5$
iv) Add – ($R_2$) , $R_5$ v) Subtract ($R_1$) + , $R_5$. **(05 Marks)**
c. Explain Logical shift and Rotate instructions with examples. **(10 Marks)**

3 a. Explain Memory mapped I/O and I/O mapped I/O. **(06 Marks)**
b. With neat diagram, explain Centralized bus arbitration and distributed bus arbitration. **(08 Marks)**
c. What is DMA? Explain the registers in a DMA interface. **(06 Marks)**

4 a. Define Interrupt. With example, explain the concept of interrupt. What are the overheads incurred in handling interrupt? **(06 Marks)**
b. With neat diagram, explain the synchronous bus transfer during an input operation. **(08 Marks)**
c. Explain the tree structure of USB with split bus operation. **(06 Marks)**

5 a. Define i) Memory latency ii) Memory Bandwidth iii) Hit – rate iv) Miss penalty. **(04 Marks)**
b. With a neat diagram, explain the internal organisation of a 2M × 8 dynamic memory chip. **(10 Marks)**
c. With a neat diagram, explain the memory hierarchy with respect to speed, size and cost. **(06 Marks)**

6 a. With neat diagrams, explain internal structure of i) Static RAM, cell and ii) ROM cell. **(08 Marks)**
b. What is Memory Mapping? With neat diagram explain
i) Direct mapping ii) Set Associative mapping. **(12 Marks)**

7 a. Perform following operations on the 5 – bit signed numbers using 2's complement representation system. Also indicate whether the overflow has occurred.
i) (-9) + (-7) ii) (+7) – (-8). **(04 Marks)**
b. With neat diagram, explain 4 – bit carry – look ahead adder. **(08 Marks)**
c. Perform multiplication for -13 and + 9. Using Booth's Algorithm. **(08 Marks)**

8 a. Design a logic circuit to perform addition / subtraction of two 'n' bit numbers X and Y. **(04 Marks)**
b. Perform the division of numbers 8 by 3 (8÷3) using Restoration Division method. **(08 Marks)**
c. With neat diagram, explain Register configuration for sequential multiplication. **(08 Marks)**

9 a. With a neat diagram, explain Single bus organisation of data path inside a processor. **(10 Marks)**
b. What are the actions required to Excuse a complete instruction. Add ($R_3$) , $R_1$. Give the control sequence for execution of instruction Add ($R_3$), $R_1$. **(10 Marks)**

10 a. With neat diagram, explain the Microprogrammed Control method for design of control unit and write the micro – routine for the instruction Branch < 0. **(10 Marks)**
b. Bring out the difference between Microprogrammed control and Hard – wired control. **(04 Marks)**
c. With neat diagram, explain 4 – Stage pipeline. **(06 Marks)**

* * * * *

CBCS SCHEME

USN | | | | | | | | | |

18CS35

# Third Semester B.E. Degree Examination, July/August 2021
## Software Engineering

Time: 3 hrs. Max. Marks: 100

*Note: Answer any FIVE full questions.*

1 a. What is software engineering? Explain its key challenges facing. (06 Marks)
b. Explain the software process activities. (06 Marks)
c. Explain software increment process model for software development. (08 Marks)

2 a. Mention and explain important categories of software products. (04 Marks)
b. Discuss essential attributes of good software. (06 Marks)
c. Explain the water fall process model and discuss the problem with water fall process model. (10 Marks)

3 a. What is object orientation? Explain the concepts of object orientation. (10 Marks)
b. What is Object Oriented Development (OOD) and Object Oriented Modeling (OOM)? (10 Marks)

4 a. Explain the OOM – class, state and interaction models in detail. (10 Marks)
b. Mention Object Orientation (OO) themes. (04 Marks)
c. Explain the three models describe a system in OOM/OOD. (06 Marks)

5 a. What is system modeling? Mention its advantages. (04 Marks)
b. Mention the various UML diagrams used in system modeling. (06 Marks)
c. Draw Classes/Associations in the MHC-PMS. (10 Marks)

6 a. Explain what is generalization is system modeling. (04 Marks)
b. Draw the generalization hierarchy for Doctor, Hospital doctor and general practioner. (06 Marks)
c. Draw state diagram of a microwave oven. (10 Marks)

7 a. Mention the important program testing goals. (06 Marks)
b. Explain the I/O model of program testing. (06 Marks)
c. Explain the software testing process. (08 Marks)

8 a. What is inspections and testing? Explain. (06 Marks)
b. Explain the stages of testing. (08 Marks)
c. Explain testing strategies-partitions (equivalence). (06 Marks)

9 a. What is project planning? Mention the project planning stages. (04 Marks)
b. What is plan-driven development and mention its – pros and cons? (06 Marks)
c. What is project planning process? Explain with diagram in detail. (10 Marks)

10 a. What is software pricing? Mention the factors affecting software pricing. (04 Marks)
b. Explain the project scheduling process. (10 Marks)
c. What is project scheduling and its activities. Also explain the milestones and deliverable in project scheduling. (06 Marks)

* * * * *

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

CBCS SCHEME

USN | | | | | | | | | | |

18CS36

# Third Semester B.E. Degree Examination, July/August 2021
## Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

**Note:** ***Answer any FIVE full questions.***





1 a. Define the following with an example for each i) Proposition ii) Tautology iii) Contradiction. **(06 Marks)**

b. Establish the validity of the argument:

$$\begin{array}{l} p \to q \\ q \to (r \wedge s) \\ \neg r \vee (\neg t \vee u) \\ \neg r \vee (\neg t \vee u) \\ \underline{p \wedge t \qquad\qquad} \\ \therefore U \end{array}$$

**(09 Marks)**

c. Determine the truth value of the following statements if the universe comprises of all non zero integers:

i) $\exists_x \exists_y [xy = 2]$
ii) $\exists_x \forall_y [xy = 2]$
iii) $\forall_x \exists_y [xy = 2]$
iv) $\exists_x \exists_y [(3x + y = 8) \wedge (2x - y) = 7]$
v) $\exists_x \exists_y [(4x + 2y = 3) \wedge (x - y = 1)]$ **(05 Marks)**

2 a. Using truth table, prove that for any three propositions p, q, r $[p \to (q \wedge r)] \Leftrightarrow [(p \to q) \wedge (p \to r)]$. **(08 Marks)**

b. Prove that for all integers 'k' and '$l$', if k and $l$ both odd, then $k + l$ is even and $kl$ is odd by direct proof. **(06 Marks)**

c. If a proposition has truth value 1, determine all truth values arguments for the primitive propositions p, r, s for which the truth value of the following compound proposition is 1.
$[q \to \{ (\neg p \vee r) \wedge \neg s\}] \wedge \{\neg s \to (\neg r \wedge q)\}$ **(06 Marks)**

3 a. Prove by mathematical induction for every positive integer 8 divides $5^n + 2 \cdot 3^{n-1} + 1$. **(06 Marks)**

b. For the Fibonacci sequences $F_0, F_1, F_2 \ldots$ Prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. **(06 Marks)**

c. Find the coefficient of :

i) $x^9 y^3$ in the expansion of $(2x - 3y)^{12}$
ii) $x^{12}$ in the expansion $x^3(1 - 2x)^{10}$ **(08 Marks)**

**4** a. Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(06 Marks)**

b. How many positive integers 'n' can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want 'n' to exceed 5,000,000. **(08 Marks)**

c. Find the number of distinct terms in the expansion of $(w + x + y + z)^{12}$. **(06 Marks)**

**5** a. i) Let $f: R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5, & \text{for } x > 0 \\ -3x + 1, & \text{for } x \leq 0 \end{cases}$$

Determine: $f\left(\frac{5}{3}\right)$, $f^{-1}(3)$, $f^{-1}([-5, 5])$ **(04 Marks)**

ii) Prove that if 30 dictionaries contain a total of 61,327 pages, then at least one of the dictionary must have at least 2045 pages. **(02 Marks)**

b. Prove that if $f : A \to B$ and $g : B \to C$ are invertible functions then $gof : A \to C$ is an invertible function and $(gof)^{-1} = f^{-1} o\, g^{-1}$. **(06 Marks)**

c. Let A = {1, 2, 3, 4, 5}. Define a relation R on $A \times A$ by $(x_1, y_1)\, R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Determine whether R is in equivalence relation on $A \times A$.

ii) Determine equivalence classes [(1, 3)], [(2, 4)]. **(08 Marks)**

**6** a. Let A = {a, b, c, d} and B = {1, 2, 3, 4, 5, 6}

i) How many functions are there from A to B? How many of these are one-to-one? How many are onto?

ii) How many functions are there from B to A? How many of these are one-to-one? How many are onto? **(06 Marks)**

b. Let A = {1, 2, 3, 4, 6, 12}. On A define the relation R by aRb if and only if "a divides b".

i) Prove that R is a partial order on A

ii) Draw the Hasse diagram

iii) Write down the matrix of relation. **(08 Marks)**

c. Define partition of a set. Give one example Let A = {a, b, c, d, e}. Consider the partition P = {{a, b}, {c, d}{e}} of A. Find the equivalent relation inducing this partition. **(06 Marks)**

**7** a. Out of 30 students in a hostel; 15 study History, 8 study Economics and 6 study Geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(06 Marks)**

b. Five teachers $T_1$, $T_2$, $T_3$, $T_4$, $T_5$ are to made class teachers for five classes $C_1$, $C_2$, $C_3$, $C_4$, $C_5$ one teacher for each class. $T_1$ and $T_2$ do not wish to become the class teachers for $C_1$ or $C_2$, $T_3$ and $T_4$ for $C_4$ or $C_5$ and $T_5$ for $C_3$ or $C_4$ or $C_5$. In how many ways can the teachers be assigned work without displeasing any teacher? **(08 Marks)**

c. Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \geq 2$. **(06 Marks)**

**8** a. Solve the recurrence relation $a_0 - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$. **(06 Marks)**

b. Let $a_n$ denote the number of n-letter sequences that can be formed using letters A, B and C, such that non terminal A has to be immediately followed by B. Find the recurrence relation for $a_n$ and solve it. **(06 Marks)**

c. Find the number of permutations of English letters which contain exactly two of the pattern car, dog, pun, byte. **(08 Marks)**

9 a. Define a complement of a simple graph. Let G be a simple graph of order n. If the size of G is 56 and size of $\overline{G}$ is 80, what is n? **(06 Marks)**

b. Prove that is every graph, the number of vertices of odd degree is even. **(08 Marks)**

c. Prove that a connected graph G remains connected after removing an edge e from G if and only if e is a part of some cycle in G. **(06 Marks)**

10 a. Define graph isomorphism and isomorphic graphs. Determine whether the following graphs are isomorphic or not. **(06 Marks)**

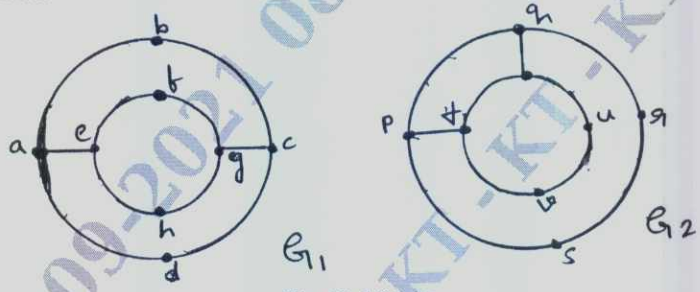


Fig.Q.10(a)

b. Prove that a tree with 'n' vertices has n – 1 edges. **(06 Marks)**

c. Define optimal prefix code. Obtain the optimal prefix code for the string ROAD is GOOD. Indicate the code. **(08 Marks)**

* * * * *

CBCS SCHEME

USN | | | | | | | | | |

18MATDIP31

# Third Semester B.E. Degree Examination, July/August 2021
## Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

***Note: Answer any FIVE full questions.***

1 a. Show that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$. **(07 Marks)**

b. Express $1-i\sqrt{3}$ in polar form and hence find its modulus and amplitude. **(06 Marks)**

c. Express $\frac{1}{1-\cos\theta+i\sin\theta}$ in the form a + ib and also find its conjugate. **(07 Marks)**

2 a. Define dot product between two vectors A and B. Find the sine of the angle between the vectors $\vec{A}=2\hat{i}-2\hat{j}+\hat{k}$ and $\vec{B}=\hat{i}-2\hat{j}+2\hat{k}$. **(07 Marks)**

b. If $\vec{A}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{B}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{C}=3\hat{i}+\hat{j}$, find the value of p such that $\vec{A}-p\vec{B}$ is perpendicular to $\vec{C}$. **(06 Marks)**

c. Find $\vec{a}\cdot(\vec{b}\times\vec{c})$, $\vec{b}\times(\vec{a}\times\vec{c})$ and $\vec{c}\cdot(\vec{a}\times\vec{b})$ where $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+2\hat{k}$, $\vec{c}=3\hat{i}-\hat{j}-\hat{k}$. **(07 Marks)**

3 a. Obtain the Maclaurin's series expansion of log(sec x) upto the terms containing $x^6$. **(07 Marks)**

b. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then using Euler's theorem, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. **(06 Marks)**

c. If u = f(x – y, y – z, z – x), prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **(07 Marks)**

4 a. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$ upto $x^4$. **(07 Marks)**

b. If $u=e^{\frac{x^2y^2}{x+y}}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3u\log u$ using Euler's theorem. **(06 Marks)**

c. If $u=\frac{yz}{x}$, $v=\frac{zx}{y}$, $w=\frac{xy}{z}$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=4$ **(07 Marks)**

5 a. A particle moves along a curve $x=3t^2$, $y=t^3-4t$, $z=3t+4$ where t is the time variable. Determine the components of velocity and acceleration vectors at t = 2 in the direction $\hat{i}-2\hat{j}+2\hat{k}$. **(07 Marks)**

b. Find the unit normal vector to the surface $xy^3z^2=4$ at the point (–1, –1, 2). **(06 Marks)**

c. Show that the vector field $\vec{F}=(2x+yz)\hat{i}+(4y+zx)\hat{j}-(6z-xy)\hat{k}$ is irrotational. Also find $\phi$ such that $\vec{F}=\nabla\phi$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Find $\operatorname{div}\vec{F}$ and $\operatorname{Curl}\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

b. If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y^4)\hat{j} - 2x^3z^2\hat{k}$ then find $\nabla\cdot\vec{F}$, $\nabla\times\vec{F}$ and $\nabla\cdot(\nabla\times\vec{F})$ at (2, −1, 0). **(06 Marks)**

c. Determine the constant 'a' such that the vector $(2x + 3y)\hat{i} + (ay - 3z)\hat{j} + (6x - 12z)\hat{k}$ is Solenoidal. **(07 Marks)**

7 a. Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x dx (n > 0)$. **(07 Marks)**

b. Evaluate $\int_0^a x^4\sqrt{a^2 - x^2}dx$. **(06 Marks)**

c. Evaluate $\int_1^5\int_1^{x^2} x(x^2 + y^2)dxdy$. **(07 Marks)**

8 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x dx\ (n > 0)$. **(07 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}dx$ **(06 Marks)**

c. Evaluate $\int_{-1}^{1}\int_0^z\int_{x-z}^{x+z}(x + y + z)dydxdz$ **(07 Marks)**

9 a. Solve $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$ **(07 Marks)**

b. Solve $\frac{dy}{dx} - y\tan x = y^2\sec x$. **(06 Marks)**

c. Solve $3x(x + y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$ **(07 Marks)**

10 a. Solve $\frac{dy}{dx} + y\cot x = \sin x$. **(07 Marks)**

b. Solve $(x + 3y - 4)dx + (3x + 9y - 2)dy = 0$ **(06 Marks)**

c. Solve $[1 + (x + y)\tan y]\frac{dy}{dx} + 1 = 0$ **(07 Marks)**

* * * * *